Noise Performance of Fast Hierarchical 3D Backprojection for Helical Cone-Beam Tomography

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***Abstract*— Existing algorithms for exact helical cone beam tomographic reconstruction involve a 3-D backprojection step, which dominates the computational cost of the algorithm. Hierarchical backprojection reduces the complexity of this step from $O(N^4)$ to $O(N^3 logN)$, greatly accelerating the reconstruction process. Here the performance of the hierarchical reconstruction is examined in the presence of noise. We demonstrate that reconstructions obtained using this method have good image quality and comparable noise performance to conventional backprojection, while providing a speedup in computation by over an order of magnitude. These properties are essential for acceptance of a fast reconstruction algorithm.**

I. Introduction

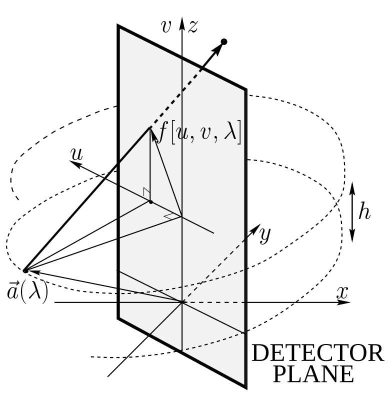

Fig. 1. Helical Cone Beam Acquisition

The data acquisition for helical cone-beam (HCB) tomography is shown in Figure 1, where the cone beam projections of an object are collected on a planar detector as the source moves in a helical trajectory about the object. Several algorithms [1] have been developed for accurate image reconstruction in HCB. These inversion formulas are of filtered backprojection form, consisting of individually filtering the cone beam projections followed by a backprojection over the image volume. This 3-D backprojection has complexity of $O(N^3P)$ for reconstruction of an $N \times N \times N$ voxel image from P projections. Generally $P = O(N)$, which results in an $O(N^4)$ operation and accounts for most of the computation in the reconstruction process. This high computational cost limits the speed that can be achieved with economically feasible implementations. Furthermore, the unfavorable $O(N^4)$ scaling limits the practicality of increasing image resolution.

Hierarchical backprojection, originally developed for 2-D parallel beam tomography [2] reduces the complexity of the backprojection operation by successive subdivision of the reconstruction area into smaller non-overlapping regions. This hierarchical decomposition has been extended to HCB geometry [3], reducing computation to $O(N^3 logN)$. The hierarchical algorithm provides large speedups for typical image sizes, and it (and its variations) have the potential to revolutionize 3D CT. Here we examine its performance in the presence of noisy data. Non-degraded noise performance will be one of the key criteria for adoption of the fast hierarchical HCB algorithm.

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II. Hierarchical Backprojection Algorithm

The hierarchical backprojection algorithms [2] are based on the principle of "divide and conquer", in which the reconstructed volume is successively divided into smaller non-overlapping volumes, and the backprojection operation is decomposed into backprojection onto the sub-volumes. This recursive decomposition is shown in Figure 2. By itself, this scheme does not yield any reduction in the number of operations required to reconstruct the volume.

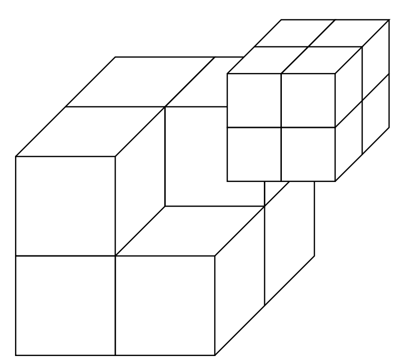


Fig. 2. Recursive Decomposition

Hierarchical backprojection takes advantage of the fact that the number of projections needed to accurately reconstruct a bandlimited subimage at the center of the source of rotation is proportional to the size of the subimage. For reconstruction of a half-size subimage, e.g. an $N/2 \times N/2 \times N/2$ voxel volume, the projection data set can be angularly decimated by a factor of 2 without any loss in reconstructed image quality. Reconstructing each of the 8 half-sized octants in the reconstruction volume with $P/2$ projections leads to a factor of 2 saving in one level of the hierarchical decomposition.

These sub-octants are off-centered regions, which prevents directly applying the sampling principle to decimate the projection data. Projections $g(u, v, \lambda)$ of the subimages are "centered" by shifting in the projection plane relative to the projection of the subregion's center $(u_c(\lambda), v_c(\lambda))$,

$$\bar{g}(u, v, \lambda) = g(u + u_c(\lambda), v + v_c(\lambda), \lambda) \tag{1}$$

The effect of the shift is to reduce the angular bandwidth, so that the centered projections $\bar{g}(u, v, \lambda)$ may be decimated without information loss. The decimated projections are then "de-centered" by shifting them back to their correct positions and then backprojected with half the computational effort.

Recursively performing this decomposition $log_2 N$ times yields a final reconstruction subvolume of 1 voxel from $O(P/N)$ projections, yielding a complexity of $O(N^2P)$ for reconstructing the entire volume. It was shown [3] that each stage of the decomposition is also $O(N^2P)$, yielding the complexity of $O(N^2 P log N) = O(N^3 log N)$, a reduction of the computational complexity of $O(N^4)$ of the conventional algorithm.

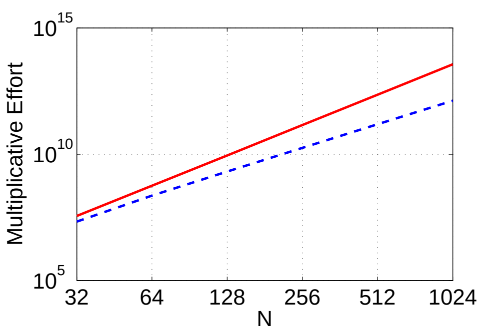


Fig. 3. Multiplicative Effort Comparison for Conventional (solid, red) and Hierarchical (dashed, blue) algorithms

The conventional and hierarchical algorithms are compared on the basis of operation count, which is a measure of the computational resources required for each reconstruction independent of hardware characteristics. The most expensive operations in these algorithms, which dominate the computational cost, will be floating point multiplies and divides. We define the total number of expensive floating point operations as the "multiplicative effort" required of each algorithm. Figure 3 compares the two for a range of $N \times N \times N$ reconstruction volumes and demonstrates the reduction in complexity between the two algorithms.

III. Simulations

The accuracy of the hierarchical algorithm is first established with a noiseless reconstruction. Simulated projection data for the standard 3D Shepp-Logan phantom was generated using analytical expressions for the cone beam projections. The data was filtered [1] followed by backprojection onto a 512^3 voxel grid using both the conventional and hierarchical backprojection algorithms. In this case the hierarchical algorithm provides a speedup in multiplicative effort by a factor of 14.8. Slices through the reconstructed volumes are shown in Figure 4. The vertical central line of the conventional (red, solid) and hierarchical (blue, dashed) is plotted in Figure 5. The two reconstructions show good quantitative agreement to within fractions of an HU in the low contrast regions. By far the primary artifact of the hierarchical reconstruction is a 3 HU overshoot at the skull edge, but even this will be insubstantial as compared to the noise in a typical CT scan.

(A) (B)

Fig. 4. Noiseless Data Reconstructions for (A) Conventional and (B) Hierarchical algorithms

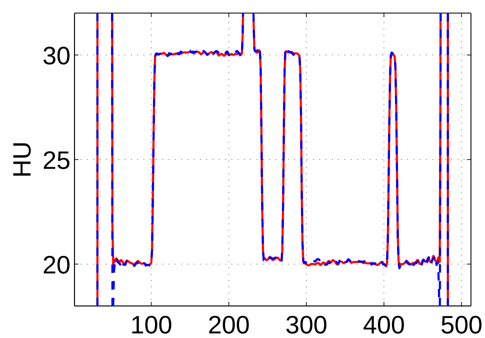
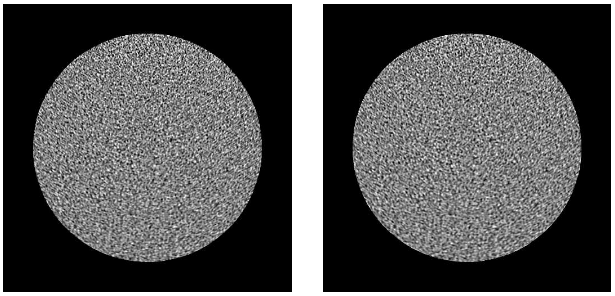


Fig. 5. Noiseless Reconstruction Slice for Conventional (solid, red) and Hierarchical (dashed, blue) algorithms

To analyze the noise performance of the hierarchical algorithm, a water cylinder mathematical phantom is used. Simulated projection data was again generated through analytical expressions, with Poisson-like noise added prior to filtering. Slices through the reconstructed object are shown in Figure 6 with a grayscale window of [-50, 50] HU. The structure of the noise is similar in both reconstructions. A section of the vertical cut through the middle of conventional (red, solid) and hierarchical (blue, dashed) is shown in Figure 7. There are



(A) (B)

Fig. 6. Noisy Data Reconstructions for (A) Conventional and (B) Hierarchical algorithms

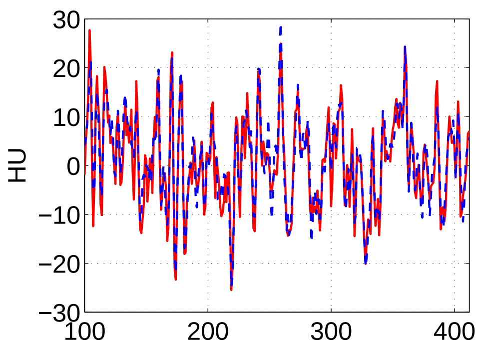


Fig. 7. Noisy Reconstruction Slice for Conventional (solid, red) and Hierarchical (dashed, blue) algorithms

some slight differences between the two, indicating hierarchical reconstruction has a somewhat different reconstruction of the noise.

For further analysis, the standard deviation of the reconstructions were calculated over a 3x3x3 sliding window, with noise variance averaged over 10 different noise realizations. The absolute difference in noise standard deviations is shown in Figure 8, windowed to the support of the phantom with grayscale window [0, 6] HU. This difference exhibits no particular structure. The noise standard deviation as a function of the distance r from the image center is shown in Figure 9. The hierarchical and conventional algorithms have similar noise properties over the image, with only small differences present.

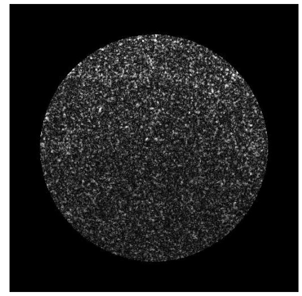


Fig. 8. Difference in Standard Deviation of Noise

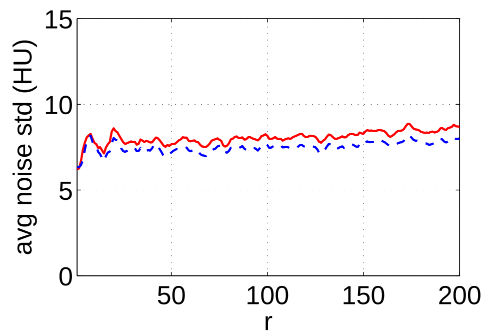


Fig. 9. Noise Distribution for Conventional (solid, red) and Hierarchical (dashed, blue) algorithms

IV. Conclusion

Hierarchical backprojection holds great potential for fast and accurate HCB reconstruction. Our numerical studies show more than an order of magnitude speedup, with no perceptible loss of accuracy and only a small change in noise properties, both of which we wish to verify on clinical data. The speedup provided by the hierarchical algorithms may prove essential for adoption of the exact helical reconstruction algorithms, which can be an order of magnitude more expensive than some approximate and artifact-prone algorithms.

References

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